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Recent Development on Mathematical Models Including Human Root Dentin and the Other Applications

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Abstract. The first aim of this paper is to survey some mathematical models including human root dentin and the other applications. It is well-known that formation, evolution, diseases and treatments of the tooth, jaws, mouth and surrounding tissues are directly related to mathematical models and their applications. In order to improve and investigate mathematical models with their algorithms, we need to understand the complex and challenging root canal system, which is an important factor for instrumentation, preparation and obturation of the tooth root canals. Therefore, root canal length, curvature, conicity, shape, and ramifications need to be evaluated in advance to enhance the success of the treatment. Therefore, to design and realize a method for analyzing the geometric characteristics of human root canals is come forward and mathematical models appear to be a suitable way to examine the geometric properties of root canals. The second aim of this paper is to provide a good reference for manufacturers of root canal instruments and for dentists to better understand the geometry of root canals as well as the limits they might have with the root canal treatment. Moreover, we give further remarks and observations on new and old mathematical models and related mathematical subjects such as the Laplace transform, the Mittag-Leffler function and the other derivative and integral formulas. Finally, we give some open questions with observations on our models.

Keywords: Laplace transform, Mittag-Leffler function, Bernstein polynomials, Bezier curves, mathematical model, human root dentin, tooth root canals.

2010 Mathematics Subject Classification: 30C10, 33E12, 44A10, 65D17, 93A30.

INTRODUCTION

In recent years, we are well aware that, by the dentists and mathematicians, numerous mathematical models and algorithms were provided in order to develop physical structures which can be used in the treatment of dental diseases and can used in the construction of tooth root canals, functional dental prosthesis and implants (*cf.* [1]–[18]; see also the other references cited therein). In this work, some basic features of some mathematical models including the geometrical properties of the tooth root canals are also discussed. In addition, the Bezier curves constructed by the Bernstein polynomials, the Mittag-Leffler function, integral transformations involving the Laplace transformation, and fractional integrals and derivatives are used in these models.

Recently, investigation of a seventh dental characteristic, quantifying the displacement of anterior teeth from the physical or native curve of each dental arch, have already found. For example, in order give the amount of displacement of the teeth, a baseline was necessary. Thus, Bezier curves, ellipse and also any other special polynomial curves are used to provide the best fit.

We now give some survey on the mathematical models for human dentin. One of them is associated with the Mittag-Leffler function $E_\alpha(t)$ (*cf.* [9]):

$$E_\alpha(t) = \sum_{k=0}^{\infty} \frac{t^k}{\Gamma(\alpha k + 1)} \tag{1}$$

where α is a positive real number.

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By using the Mittag-Leffler function with the Laplace transform, fractional derivative including the Riemann-Liouville form:

$$\frac{d^{\alpha}}{dt^{\alpha}} f(t) = \frac{d}{dt}\left(\frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{f(\tau)\, d\tau}{(t-\tau)^{\alpha}} \right),$$

where Γ is the Euler gamma function, and also with help of experimental results related to thermodynamical restrictions, Petrovic *et al*. [10] constructed a mathematical model which predicts creep in human root dentin on the Piola–Kirchhoff stress. They gave some observations and comment for their method, contained results and their significance.

The second mathematical model is depend on geometry shapes of the human dentin and their parametric features. That is, Figure 1 gives us another mathematical model which is so-called parametric model for the description of the root canal. In [2], by using curvature, length, shape, and ramifications, Dannemann *et al*. gave root canal geometry model, including the elliptical model, the 1-circle model, and the 3-circle model, which is an important factor for instrumentation and preparation of the canals. These geometric parameters, curved shapes, characteristics, and structures are used to successfully evaluate treatment of human root canals. For detail information related to this model, see the work [2].

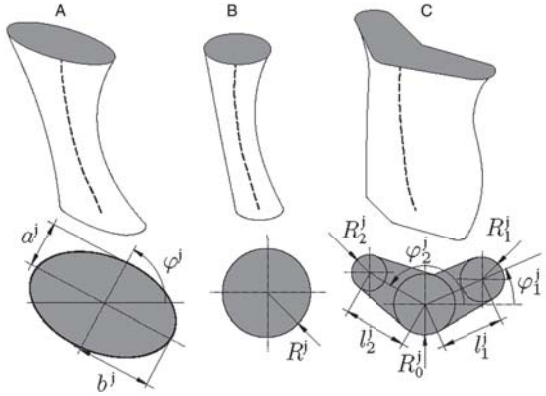


FIGURE 1: The scheme of parametric models, examined by Dannemann *et al*., for the description of the root canal with the central lines and the top cross section (gray). (A) The elliptic model, (B) The circle model, and (C) The 3-circle model (*cf*. [2]). For further details related to these models, see the work [2] of Dannemann *et al*.

The third mathematical model is related to not only the Bezier curves, but also Non-Uniform Rational B-Spline (NURBS) which are related to the Bezier curves and the Bernstein basis functions. Hieu *et al*. [6] gave some methods associated with these curves. In addition, it is well-known that there exist various unification of the Bezier curves and NURBS. For instance, in [17], the second author constructed the q-Bezier type curves $B_n(x : q)$ with control points $P_0, \cdots, P_n$ as follows:

$$B_n(x : q) = \sum_{k=0}^{n} P_k \mathfrak{b}_k^n(x; q). \tag{2}$$

where $x \in [0, 1]; k = 0, \ldots, n$ and $\mathfrak{b}_k^n(x; q)$ denotes the q-Bernstein basis functions (see [17]). Note that in the special case when $q \to 1$, the q-Bezier type curves and the q-Bernstein basis functions are reduced to the classical Bezier curves and the classical Bernstein basis, respectively (*cf*. [3], [5]). Thus, the classical Bernstein basis functions are given by (*cf*. [3], [4], [8], [14], [15]; and also, for detail see [7], [11], [12], [13]):

$$\frac{1}{k!}(xt)^k e^{t(1-x)} = \sum_{n=k}^{\infty} B_k^n(x) \frac{t^n}{n!},$$

where

$$B_k^n(x) = \begin{pmatrix} n \\ k \end{pmatrix} x^k (1-x)^{n-k},$$

and

$$\left(\begin{array}{c} n \\ k \end{array} \right) = \frac{n!}{k!(n-k)!}.$$

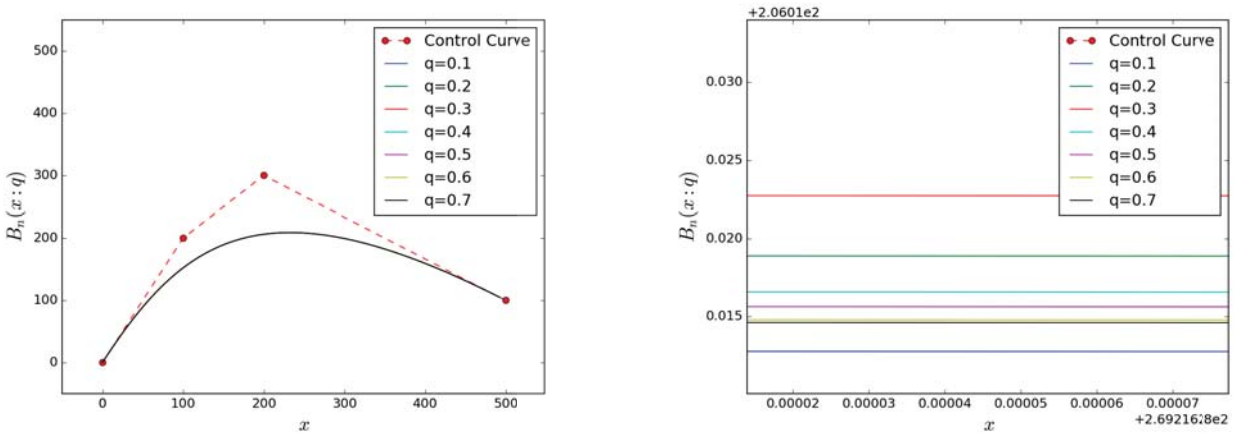


FIGURE 2: q-Bezier type curves obtained varying q from 0.1 to 0.7 and control points $(0,0)$, $(100,200)$, $(200,300)$ and $(500,100)$ and its closer (*cf.* [17, p. 165]).

REMARKS AND OBSERVATIONS

In this article, some special mathematical models, which are known up to now, are given with their brief properties. In this section, some special questions including these models will be raised.

In [16] and [17], the second author constructed two kind of Bezier type curves. In [17], he also mentioned that the q-Bernstein basis functions might be affected by selecting different q values on the shape of the curves. In the light of this aspect, one may develop the NURBS modeling which are applied on the radiographical images of teeth, to more effective model so-called q-NURBS type modelling including q-Bezier type curves. That is, by applying q-Bezier type curves given in Figure 2, one can modify and unify the idea given in Figure 3. Maybe, this is a better solution for the NURBS modeling for radiographical images of teeth.

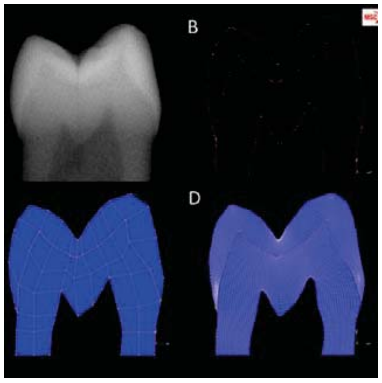


FIGURE 3: An example of the NURBS modeling for radiographical images of teeth (*cf.* [18]). For further details related to this model, see the work [18] of Soares *et al*.

In conclusion, some open questions, which are given in below section, are raised up in the light of the above observations and survey.

Open questions

Can new models with human root dentin, mouth and other jaw problems be constructed? Is it possible to develop a method to compare existing models with these new models?

While new models involving human root dentin, mouth, and other jaw problems are being constructed, which of the Bezier curves, geometric objects, and special mathematical functions are more optimal, effective, functional, economic and useful than well-known mathematical models?

As a result, the spirit of this paper leads the way of potential methods and models for the researchers who study not only endodontics, but also related areas such as medicine, biology and the others. Moreover, when the solution of the aforementioned open questions are investigated and solved, these solutions contribute to the theory of mathematical modelling on dentin.

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