LINEARIZED STABILITY ANALYSIS OF SPACE FRAMES USING A DEGENERATED BEAM FINITE ELEMENT

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ABSTRACT

In this study, stability analysis of space frames using degenerated 3D beam finite elements is investigated. For this purpose, a degenerated beam element having offset property is used by modifying its torsional rigidity and using an additional warping degree of freedom. Comparisons with analytical and numerical results available in the literature show that the selected element is capable of accurately calculating the lateral torsional buckling parameters of beams having bi-symmetrical thin-walled open sections. Additionally, a sample frame problem is solved and the numerical results obtained are presented in comparison with the results found in the literature. The results obtained show that, the global buckling parameters of space frames having bi-symmetrical open cross-sections can be determined to a sufficient accuracy using the developed beam finite element.

Keywords: thin-walled open section, frame, buckling, finite element method

UZAY ÇERÇEVELERİN DEJENERE BİR KİRİŞ SONLU ELEMANIYLA LİNEER BURKULMA ANALİZİ

ÖZET

Bu çalışmada, uzay çerçeve toptan burkulma analizinde dejenere kiriş sonlu elemanlarının kullanımı incelenmiştir. Bu maksatla, öteleme özelliğine sahip bir dejenere kiriş elemanı burulma rijitliği modifiye edilerek ve çarpılma için bir serbestlik ilave edilerek kullanılmıştır. Literatürde bulunan analitik ve sayısal neticelerle yapılan karşılaştırmalar seçilen eleman ile çift-simetrik açık kesitli kirişlerin yanal burkulma parametrelerinin hassas bir şekilde hesaplanabildiğini göstermektedir. Ayrıca, bir çerçeve burkulması problemi için elde edilmiş sayısal neticeler literatürde bulunan neticelerle karşılaştırılmalı olarak sunulmuştur. Elde edilen neticeler geliştirlimiş kiriş elemanı ile çift simetrik açık kesitli elemanlardan oluşan uzay çerçevelerin global burkulma parametrelerinin hassas bir şekilde belirlenebildiğini göstermektedir.

Anahtar Kelimeler: ince cidarlı açık kesit, çerçeve, burkulma, sonlu elemanlar yöntemi

1. INTRODUCTION

In this study, stability analysis of space frames using degenerated 3D beam finite elements is investigated. The main motivation to use a degenerated element is to be able to obtain results not limited with the assumptions of a particular beam theory.

A degenerated shear deformable beam element having nonlinear analysis capability was developed by Balourchi and Bathe (1981), which can only be used to model beams having rectangular cross-section. Similar degenerated beam super-elements have also been developed and used by Ferguson and Clark (1979), Kanoknukulchai and Sivakumar (1987), Yoon and Lee (2012) to be able to consider more general cross-sections and/or the influence of warping.

In the present study, a standard degenerated beam super-element is improved by modifying the displacement field to be able to consider warping and this modified element is used to obtain the numerical results presented here for 3D frames having bi-symmetrical I beam sections.

Comparisons with analytical and numerical results available in the literature are made and the results obtained are presented in tabular and graphical form.

2. NUMERICAL ANALYSIS PROCEDURE

The degenerated 3D beam element used in this study is based on the element developed by Ferguson and Clark (1979). It is assumed that the normals to the mid-surface remain straight and thus the element is first order shear deformable. The geometry of a beam segment is shown in Fig.1.

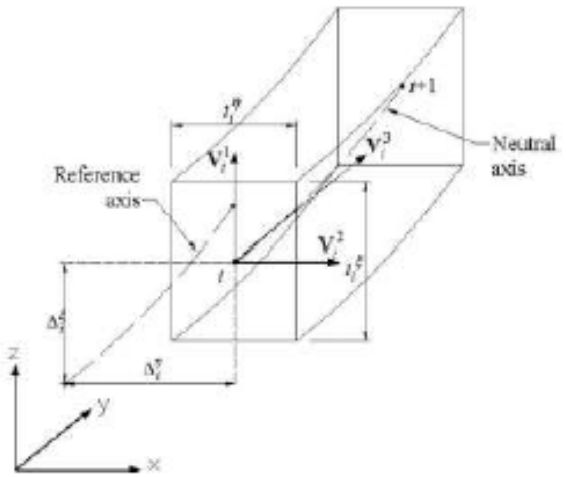


Figure 1. Geometry of a beam segment

There're 7 nodal variables at each node, i.e. the translations in x, y, z directions and the rotations about $\mathbf{V}_i^1, \mathbf{V}_i^2, \mathbf{V}_i^3$. The nodal displacement vector is denoted by $\boldsymbol{\delta}^e$ which is given below.

$$\boldsymbol{\delta}^e = \begin{Bmatrix} \boldsymbol{\delta}^1 & \boldsymbol{\delta}^2 & \boldsymbol{\delta}^3 \end{Bmatrix}^T, \text{ where } \boldsymbol{\delta}^i = \{u_i \quad v_i \quad w_i \quad \alpha_i \quad \beta_i \quad \gamma_i \quad \varphi_i\}^T$$

where φ_i denotes the warping degree of freedom. The displacements are;

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \sum_{i=1}^{3} S_i \begin{Bmatrix} u_i + u_{i,w} \\ v_i + v_{i,w} \\ w_i + w_{i,w} \end{Bmatrix} + \sum_{i=1}^{3} S_i \left(\xi \frac{t_i^{\xi}}{2} + \Delta_i^{\xi} \right) [\boldsymbol{V}_i^2, -\boldsymbol{V}_i^3] \begin{Bmatrix} \gamma_i \\ \beta_i \end{Bmatrix} + \sum_{i=1}^{3} S_i \left(\eta \frac{t_i^{\eta}}{2} + \Delta_i^{\eta} \right) [-\boldsymbol{V}_i^1, \boldsymbol{V}_i^3] \begin{Bmatrix} \gamma_i \\ \alpha_i \end{Bmatrix}$$

$$\begin{Bmatrix} u_{,w} \\ v_{,w} \\ w_{,w} \end{Bmatrix} = \sum_{i=1}^{3} S_i \begin{Bmatrix} u_{i,w} \\ v_{i,w} \\ w_{i,w} \end{Bmatrix} \qquad (1)$$

The magnitude of the warping displacement is assumed to be given by the product of two functions: the normalized section warping function and the warping amplitude. Modified form of the warping related strain terms defined by Gunnlaugsson and Pedersen (1982) are used in this study; see Cagdas (2022).

The other important modification is the torsional rigidity modification. The degenerated beam element used is not recommended as a torsional spring if the cross-section deviates significantly from the square. Therefore, torsional rigidity is calculated analytically using the following formula presented by Trahair (2017);

$$J = \frac{bt^3}{3} \left(1 - 0.674 \frac{t}{b} + 0.096 \frac{t^2}{b^2} \right) \qquad (2)$$

Also, a superparametric-degenerated shell element developed by Cagdas and Adali (2012) is used in this study for comparison purposes.

3. NUMERICAL RESULTS

3.1. Verification Problem

First, a 2 storey 1 bay frame having rectangular homogeneous isotropic members is analyzed using the 3D beam element and comparison is made with the numerical results obtained using a 2D shear deformable plane beam element.

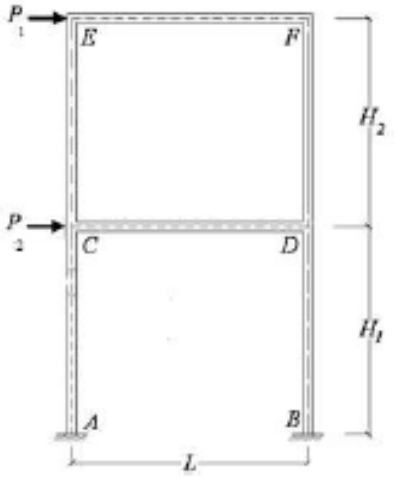


Figure 2. Geometry and loading for the verification problem

Table 1. Linear static analysis results for a homogeneous isotropic frame with E=2.1×10^{11} N/m^2, □=0, b=0.1 m, h=0.2 m, L=5 m, H_1=H_2=3 m subject to $R_1 = P_2$=10^5 N.

Variable	(2D) 6 el.	(3D) 6 el.	(2D) 12 el.	(3D) 12 el.
δ_C (cm)	3.055	3.076	3.055	3.076
δ_E (cm)	6.146	6.183	6.146	6.184
ψ_C (rad)	0.00964	0.00967	0.00964	0.00967
ψ_E (rad)	0.00562	0.00564	0.00562	0.00564

Table 2. Buckling parameters for a homogeneous isotropic frame with E=2.1×10^{11} N/m^2, □=0, b=0.1 m, h=0.2 m, L=5 m, H_1=H_2=3 m. (in-plane buckling)

Loading	(2D) 6 el.	(3D) 6 el.	(2D) 12 el.	(3D) 12 el.
$R_1 = P_2$=10^5 N	63.5657	63.1219	61.9128	61.6173
R_1=10^5 N	123.7640	125.6852	123.002	122.3477

3.2. Lateral Trosional Buckling (LTB) Problem 1

A LTB problem, studied by Demirhan et al. (2020), is solved to verify the element. A cantilever beam under the action of a tip transverse load is analyzed and the results are presented in comparison with the reference results and the numerical results obtained using an 8x3 mesh of 3D beam elements and a 2D shell finite element mesh composed of 480 shell elements. The beam cross-section for IPE160 is approximated by an I beam having total height h=160 mm, h_w=web height 145.2 mm, flange width b=82 mm, flange thickness t_f=7.4 mm, web thickness t_w=5 mm.

The numerical results obtained for the 1st problem are presented in Table 3 in comparison with the results presented by Demirhan *et al.* (2020), and the results obtained using a 2D shell finite element model having 480 degenerated shell elements; see Cagdas and Adali (2012). It can be observed from Table 3 that the LTB loads obtained using an 8x3 mesh of 3D beam elements is in excellent agreement with the reference results obtained using software package Abaqus and the 2D shell model.

Table 3. LTB loads (N) for LTB problem 1

L(m)	Specimen	Load position	Beam F.E. model (8x3 elements)	Linear buckling load (Abaqus) Demirhan et al. (2020)	Shell FE Model (480 elem.)
2,75	1	Upper flange	**8787,8**	8980,3	8942,7
2,5	2	Upper flange	**10400**	10593	10563,9
2,25	3	Upper flange	**12499**	12679	12655,2
2,75	7	CG	**14324**	14354	14354,0
2,5	8	CG	**18045**	17852	18050,3
2,225	9	CG	**23342**	22621	23280,0

The buckling mode shapes for L=2.75 m and for upper flange loading obtained using the shell and beam models are shown in Figures 3(a) and 3(b), respectively. The normal force distribution in axial direction obtained using the shell model is shown in Fig. 4.

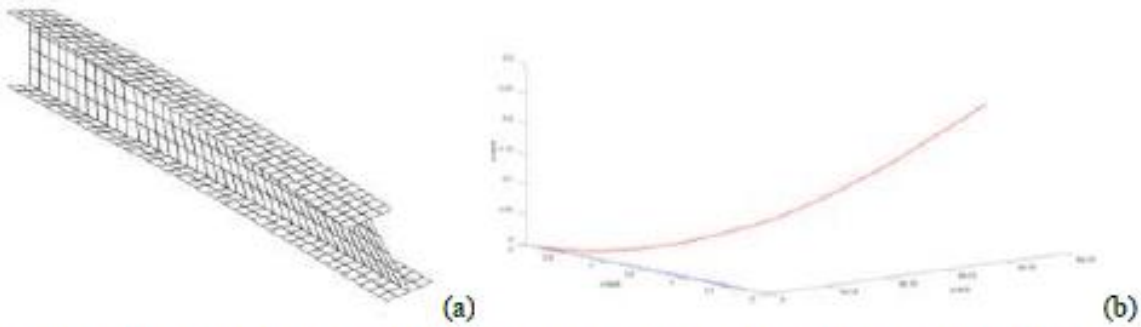


Figure 3. Buckling mode shape for L=2.75 m, upper flange loading obtained by (a) the shell model (b) the beam model

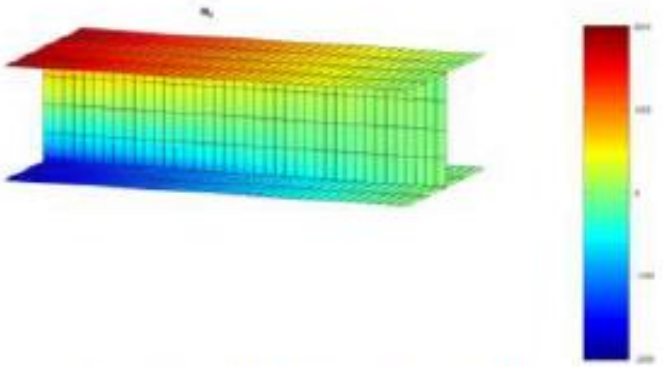


Figure 4. Normal force distribution for P=1000 N.

3.3. LTB Problem 2

A portal frame under 2 point loads as shown in Fig. 5 is analyzed and the results obtained are presented in Table 4 in comparison with the reference results. Note that, the beam cross-section for W200x59 is approximated by an I beam having total height h=195.8 mm, h_w=web height 167.4 mm, flange thickness t_f=14.2 mm, web thickness t_w=5 mm, and flange width b=204 mm.

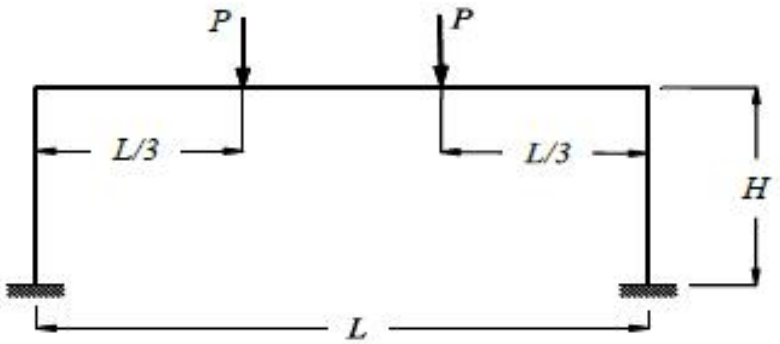


Figure 5. Portal frame geometry and loading

The numerical results presented in Table 4 are in good agreement is with the results presented by Wu and Mohareb (2013). However, the shell model presented by Zino (2010) yields a lower LTB load.

Table 4. Numerical results for problem 2

				P_{cr3}	P_{cr5}	
H (m)	L(m)	Abaqus Zino(2010)	Joint element Wu Muharib (2013)	Wu Muharib (2013)	Wu,Mohareb (2013)	Present study
4	6	219,7	236,1	247	268,5	**271,56**

The buckling mode shape is shown below in Fig.6.

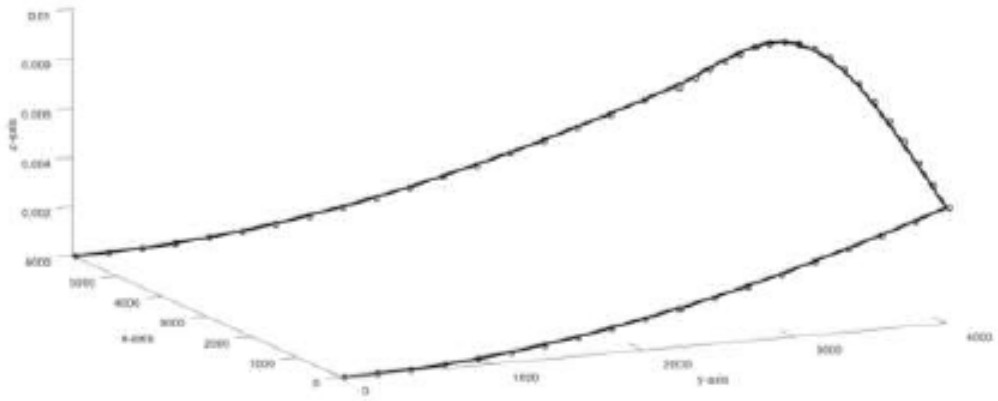


Figure 6. Buckling mode shape for L=6m, H=4m.

CONCLUSIONS

Stability analysis of space frames using degenerated 3D beam finite elements is studied and a modified degenerated beam element is used for this purpose.

After verifying the element for linear static analysis and linearized stability analysis, 2 lateral torsional buckling problems are solved. The results presented for the LTB problems are found out to be in very good agreement with the reference results obtained by using beam theory based shear deformable elements.

Extension of the present study to asymmetric and monosymmetric beam-column stability problems is also possible but this requires uncoupling of displacements and rotations. The element may also be used to model laminated composite beams by making use of its offset property. The related work is under progress.

It should also be stated that it is relatively easy to extend the capability of degenerated element used here for both geometrically and materially nonlinear analysis as the formulation does not require any beam theory.

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