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Analysis of δ Librae including *Hipparcos* astrometry

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ABSTRACT

New spectroscopy of the classical Algol system δ Lib, combined with high-quality optical and infrared photometry, provides the basis for a good understanding of the close binary system's main parameters. Detailed analysis of the photometry reveals the significant role of a third light source, pointing to the existence of a companion to the eclipsing system of mass $\sim 1\,M_{\odot}$.

We review the methodology of applying high-accuracy positional information, available from the *Hipparcos* Intermediate Astrometric Data archive, to stars that may have such companions. Analysis of the astrometry of δ Lib also points to a third star similar to the one already identified by Worek from radial-velocity data, although with slightly revised parameters. O–C data do not contradict this, but their general precision (while confirming the close pair's Algol status) fails to allow a decision on the third orbit parameters: Worek's or revised ones. Taking the photometry, spectroscopy and astrometry together, however, the existence of a third star of comparable mass to the Sun, as a relatively close companion to the eclipsing binary ($\sim$4 au), is confirmed.

Key words: binaries: close – stars: individual: δ Lib.

1 INTRODUCTION

At its distance of $\sim$95 pc, δ Librae (= HR 5586 = HD 132742 = HIP 73473) is one of the nearest Algol systems. It is a 'classical' example of the type, made up of A0V + K0IV stars with V magnitude $\sim$4.9 and orbital period close to 2.33 d. Algols typically consist of an early-type near-MS primary and a cool 'semidetached' secondary that essentially fills to overflowing its Roche surface of limiting dynamic stability (cf. Plavec 1973). A precursor to the present work (Budding et al. 2005a) compared δ Lib with the similar binaries V505 Sgr and β Per. All these relatively near Algols have been found to be in small multiple systems with three or more stars. Worek (2001), for example, reported the likely presence of a relatively near third component in δ Lib, with a configuration quite similar to β Per. The Washington Visual Double Star Catalogue (Worley & Douglass 1996) reports three other measured faint companions (δ Lib B, C and D).

The O–C diagram formed by the times of minima of δ Lib shows an upwards parabolic trend, on to which there is a suggestion of some additional cyclic variation (see Kreiner, Kim & Nha 2001; Worek 2001; Budding et al. 2005a). The upwards parabola should be the result of the net effect of mass transfer (from the less massive secondary to the primary) and perhaps mass loss from the system. It is clear in the diagram that the mass transfer effect should be dominant, with conservation of angular momentum then entailing a steady separation of components. Budding et al. (2005a) quantified the predominating parabolic variation for δ Lib

$$\mathrm{HJD}_{\mathrm{min}I} = 244\,2960.705 + 2.327\,354\,16E + 4.44 \times 10^{-10}E^2. \quad (1)$$

With Tomkin's (1978) spectroscopic mass ratio (0.345 ± 0.012), a 'conservative' mass transfer turned out as $\sim 1.1 \times 10^{-7}\,M_{\odot}$ per year. This is in broad keeping with the recession stage of the Case B model of Algol evolution, where the loser's original mass was a few times that of the Sun (cf. e.g. Özdemir et al. 2001). For δ Lib, we can examine residuals from the main parabolic trend for further effects in view of the possible additional components in the system (Worek 1998; Lazaro, Arevalo & Claret 2002; Budding et al. 2005a). In their published information, both *Hipparcos* and Tycho surveys indicated δ Lib to have well-defined positional parameters, suggesting the parallax, at least, should not be greatly at variance with the published mean. The 'eclipse method' parallax given in table 1 of Budding, Sullivan & Rhodes (2005b) is, however, 9 per cent greater than the *Hipparcos* value. It could be that extra stars compromise the parallax determination: in the present paper, we set out a systematic procedure to check the evidence (Section 3).

2 PHOTOMETRY AND SPECTROSCOPY

The photometric variability of δ Lib was discovered by Schmidt (1865), and a photoelectric light curve appeared as long ago as 1920 (Stebbins 1934). Koch (1962) produced a set of high-quality

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light curves using filters close to the *UBV* system.[1] Radial-velocity (hereafter RV) measurements of δ Lib led Schlesinger (1909) to note the rotational effect, later discussed more thoroughly by Rossiter (1924) and McLaughlin (1924) and usually associated with those names. Later spectroscopy has tended to assign an A0V classification to the primary (Roman 1956; Sahade & Hernandez 1963; Tomkin 1978), although Koch's photometry suggested a late B spectral type. Tomkin (1978) identified secondary features in the spectrum, whose type he inferred to be early G. He measured RVs of both stars, thereby deriving the mass ratio mentioned above, citing masses of 4.9 and 1.7 $M_\odot$, to within errors of about 5 per cent. These masses seem large. The primary, for example, should be of earlier type than A0, if that star is, as usual for Algol primaries, reasonably close to the Main Sequence.

2.1 Light-curve analysis

A detailed study of the light curves of δ Lib was carried out by Lazaro et al. (2002), who included their own new light curves in the *J*, *H* and *K* bands. These good quality data should permit well-defined photometric parameters for both stars, given that the secondary minimum becomes much deeper in the infrared. Combining this with available spectroscopy allows absolute parameters to be checked. All these photometric data were independently analysed using an updated ILOT technique, as reported by Budding et al. (2005a). These fittings were also checked with other recent eclipsing binary light-curve analysis programs. Combined results are listed in Table 1 (using standard symbols), and a typical set of corresponding curve-fits are shown in Fig. 1. Formal parameter accuracy, in Table 1, is approximately reflected in the bracketed digits placed alongside the adopted values.

With the fractional luminosities of Table 1, the primary's *V* magnitude becomes 5.05. Using the *Hipparcos* distance of 93 pc, we find the absolute magnitude as $M_V = 0.16$. This appears significantly brighter than typical Main Sequence stars of around A0 spectral type. That the primary is not too far from the standard star Vega in temperature can be confirmed by calculating its magnitudes in the various bands, using corrected quadrature measures from Koch (1962) and Lazaro et al. (2002). We give the results in Table 2, together with those of the secondary and the putative third star arising from the present analysis.

Lazaro et al. (2002) remarked that their solutions were not entirely self-consistent across the full range of wavelengths studied. They suggested various possible sources of difficulty, including the role of extra absorption from circumstellar matter or the presence of a third body, although their fitting procedure did not set this as a free parameter, as in Table 1. The presented solutions, given that they have high determinacy and combine three separate approaches with different programs, give persuasive evidence about a third star. The magnitudes of Table 2 point to the third star being a mid-to-late F-type object, with a feasible (MS) mass of about $1.1 \pm 0.1\,M_\odot$ (Budding & Demircan 2006).

The somewhat bluer colour of the primary relative to Vega across the whole wavelength range, using the approximate proportionality to reciprocal temperature difference for slight changes of colour (Budding & Demircan 2006), indicates a temperature of about 5 per cent higher than Vega's 9550 (Castelli & Kurucz 1994), i.e. 10 000 K. We had adopted the value 9800 K, close to the preferred one of Lazaro et al. (9760 K), in preparing the auxiliary parameters (e.g. gravity-brightening and reradiation coefficients) of Table 1, but these are not changed significantly by a shift to 10 000 K, adopted for the later fittings.

[1] We have followed Koch's designation of his near-*V* filter as *Y*.

Table 1. Photometric curve-fitting results for δ Lib using ILOT, WD and FOTEL programs.

Parameters	ILOT	WD method	FOTEL
T_h (K)	9800	9800	9800
T_c (K)	5200	4970(20)	5200
q	0.383	0.383	0.383
L_1-U	0.916(5)	0.955(7)	0.840(30)
L_1-B	0.904(5)	0.937(7)	0.827(21)
L_1-Y	0.858(5)	0.891(7)	0.795(19)
L_1-J	0.679(3)	0.701(5)	0.677(9)
L_1-H	0.609(7)	–	0.546(8)
L_1-K	0.583(5)	0.609(4)	0.578(6)
L_3-U	0.041(5)	0.024(8)	0.108(26)
L_3-B	0.043(4)	0.027(8)	0.118(18)
L_3-Y	0.050(5)	0.037(8)	0.125(16)
L_3-J	0.070(4)	0.070(7)	0.139(8)
L_3-H	0.046(8)	–	0.187(8)
L_3-K	0.048(6)	0.075(7)	0.152(5)
r_h (mean)	0.282(7)	0.304(1)	0.315(13)
r_c (mean)	0.309(7)	0.299(2)	0.320(5)
$i(\circ)$	80.0 (3)	80.8(2)	82.0 (1)
Error meas.	$\chi^2/\nu = 1.05$ $\Delta l = 0.0065$	$\Sigma = 0.010$	

Note: The ILOT solutions are a second generation, in which the mean of geometrical parameters from a first set of seven-element solutions was used in fittings to optimize only the relative mean luminosities. Note also that the latest WD user code does not include a direct facility for handling the *H* filter. The WD error formulae assume independent parameters, so they should, in principle, have smaller numerical values than the ILOT ones. The representative error measure Σ given for the WD program is a sum of squares of residuals. There are about 200 points, on average, in each of the light curves, so an observational error of ~0.007 (cf. Δl for the ILOT fitting) should yield $\Sigma \sim 0.01$.

2.2 Spectroscopic observations

Lazaro et al.'s discussion indicated the desirability of further spectroscopy, both to check the previous analysis of Tomkin (1978) and to provide evidence for their remarks on unresolved photometric issues.

Two sets of spectroscopic observations were used in the present study. The first one was obtained in 1996 March and April, and 1997 April, with the Reticon detector at the coudé spectrograph of the Ondřejov 2-m reflector. The second set comes from the 2-m Ritchey Chretien telescope of the Rozhen Observatory in Southern Bulgaria from 2003 June to August. A list of all used spectra is given in Table 3, where the HJD of observation corresponds to the mid-exposure times. The phases ϕ of the close orbit were computed using the ephemeris (1), and they agree with those following from Kreiner et al. (2001) to within 0.002. The phases ϕ' of the wide orbit were calculated with the period 1008.8 d and epoch HJD 244 3708.88.

The Rozhen spectra were obtained using a coudé spectrograph with a Photometrics™ AT200 camera, equipped with an SITe™ SI003AB 1024 × 1024 CCD. The Ondřejov spectra cover a spectral range from about 6300 to 6750 Å with wavelength resolution of about 0.25 Å pixel^{-1}, while the Rozhen spectra span approximately from 6500 to 6600 Å with a resolution of 0.1 Å pixel^{-1}.

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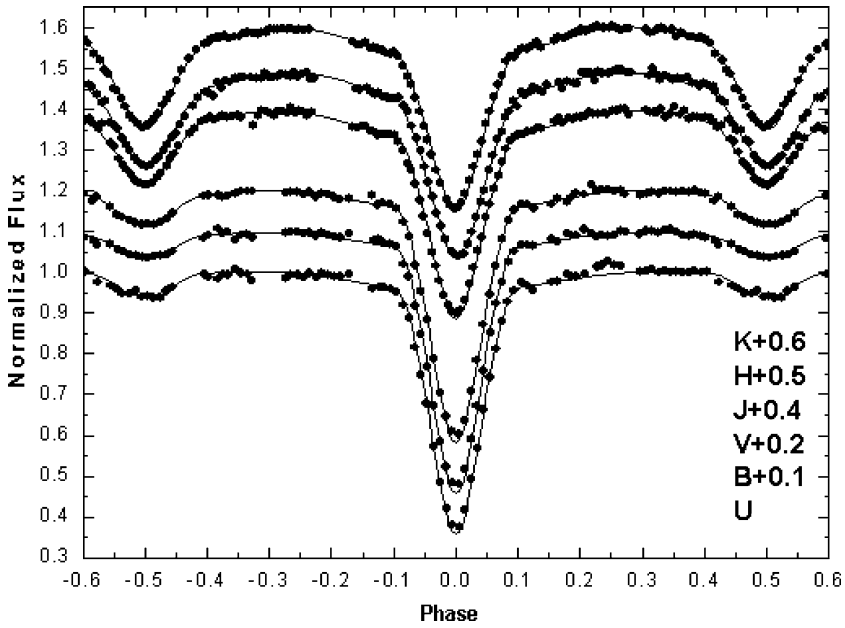


Figure 1. *UBYJHK* light curves and fittings for δ Lib.

Table 2. Magnitudes of stars in the δ Lib (ABC) system.

	U	*B*	*V*	*J*	*H*	*K*	Err.
m_1	4.86	4.99	5.05	5.04	5.12	5.07	0.02
m_2	9.11	8.86	8.02	6.48	5.74	5.92	0.1
m_3	8.21	8.25	8.06	7.39	7.92	7.21	0.1

Table 3. Spectroscopic observations of δ Lib (1996, 1997 and 2003).

No.	HJD −245 0000	Exp. (s)	ϕ	ϕ'	RV_1 (km s^{-1})	RV_2 (km s^{-1})
			Ondřejov			
1	0152.6312	9927	0.173	0.388	−71.43	188.31
2	0170.5062	10077	0.853	0.405	69.25	−179.11
3	0174.5554	13045	0.593	0.409	43.47	−138.60
4	0189.4989	12960	0.014	0.424	−15.98	17.10
5	0190.4797	7352	0.435	0.425	−41.63	108.70
6	0191.4217	10150	0.840	0.426	65.14	−185.36
7	0540.5876	13120	0.867	0.772	64.50	−160.40
8	0548.5951	8306	0.308	0.780	−81.68	198.86
			Rozhen			
9	2805.3365	960	0.966	0.017	40.03	−33.36
10	2809.3313	713	0.682	0.021	84.15	−191.19
11	2830.3380	1211	0.708	0.042	94.02	−224.48
12	2831.3231	1200	0.132	0.043	−51.15	158.93
13	2834.3384	978	0.427	0.046	−30.80	102.22
14	2863.2630	720	0.855	0.075	54.66	−164.97
15	2863.3430	764	0.890	0.075	47.19	−137.47
16	2864.2639	479	0.285	0.076	−80.10	199.81
17	2864.3477	600	0.321	0.076	−72.00	198.34

Each image was dealt with according to normal procedures of bias and dark subtraction, flat-field division, rectification of the background continuum and wavelength calibration. In particular, the Ondřejov data have been processed using the SPEFO code (cf. Horn et al. 1996; Škoda 1996).

A small eccentricity from the RV-curve solutions of the close orbit of δ Lib was reported from different data sets (0.048 by Sahade & Hernandez 1963, 0.069 by Tomkin 1978 or 0.02 by Worek 2001), and with quite different longitudes of periastron (153°, 115.5° or 314°, respectively). In spite of circularization processes, some eccentricity can be induced by interaction with a third companion in a wide eccentric orbit (cf. Eggleton & Kiseleva-Eggleton 2001, 2002). Some apparent eccentricity can also be due to orbital line-profile distortions caused by proximity effects and/or the circumstellar matter in the system (cf. e.g. van Hamme & Wilson 1985 or Antokhina, Cherepashchuk & Shimanskii 2005).

In the case of δ Lib, to verify how reliable is the evidence for eccentricity in the observational data, we re-analysed the RVs by Worek (2001) and Tomkin (1978) using the code FOTEL (Hadrava 2004a). This code enables a direct fit of all individual source velocities (taking into account the light-time or other comparable effects). Keeping the quadratic ephemeris (1), we arrived at a slightly different solution and errors. [We have also amended, both in Worek (2001) and in the erratum PASP 113, p. 1157, the obviously wrong sign of the velocity at HJD. . .5832.927).

The best fit to Worek's RVs, with residual scatter of velocities 4.2 km s^{-1}, gives $e = 0.030 \pm 0.012$ and $\omega = 335 \pm 23°$; however, what is significant and quite common for such small eccentricities is the correlation of ω and the time of periastron T that is as high as 0.9996. Moreover, both these parameters are correlated with K at the level 0.998, and with e at 0.65. One should be aware that light- and RV-curve fitting are non-linear problems, so that the covariance matrix obtained by linearization at the best fit cannot describe the behaviour of the minimized function over a wide range of significantly correlated parameters. It means that the eccentricity is practically below the resolution of the data. Fixing it equal to zero, a best fit can be found with residual scatter 4.4 km s^{-1}. The orbital parameters of the wide orbit are also much less certain than they seem to be from the errors given by Worek (2001). If all parameters are converged simultaneously, ω' and $T'_{\rm per}$ of the wide orbit are anticorrelated at −0.9999, while the error of e' (correlated with $T'_{\rm per}$ at 0.999) is larger than its value.

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The difference of errors between Worek's fit to the γ-velocities at different observational seasons and our direct fit to the RVs is partly due to the two-step nature of the former procedure, which neglects the errors of the seasonal γs. The same problem will occur with any RV-curve solution: usually published errors of orbital parameters are in fact errors of the mathematical fit to input data values, and not of the real values of physical quantities. Even in the case of a simple binary, formal errors can be significantly smaller if one solves a RV-curve constructed from mean RVs of each exposure, rather than directly from the RVs of different lines in each spectrum, the scatter of which involves at least some uncertainties of the measurement (but not all possible systematic errors).

An important source of errors for the wide orbit is poor phase coverage. To be able to solve the six unknown parameters of the wide orbit, Worek used, in addition to his own four new γs, one point published by Tomkin and three others from earlier epochs. Concerning the latter, he wrote, '... the papers tabulating these RVs give little or no details as to how the velocities might have been adjusted for any zero-point errors'. In our Solution A, given in Table 4, we used Worek's (2001) more precise RVs of the primary and Tomkin's (1978) RVs for both primary and secondary, which have a mean scatter of about 12 km s^{-1}, but which are needed to determine the mass ratio. To decrease the number of free parameters here, we have adopted P', e' and ω' as the values given by Worek.

In our present study, the spectra were measured using a new version of the code KOREL, i.e. by the method of Fourier disentangling (Hadrava 1995, 2004b), which directly yields the orbital parameters and decomposed spectra of the individual components. The Hα region has been disentangled from both the Ondřejov and the Rozhen sets of spectra. Because the Ondřejov spectra show appreciable telluric lines, the telluric component was always involved in the disentangling, with strength factors determined separately from region 6510 to 6527 Å for the Ondřejov spectra, and with a fixed negative value for the Rozhen spectra, in which telluric lines are not visible (cf. Hadrava 2004b).

Attempts were made to decompose the Hα region freely into one corresponding to a triple stellar system. However, the decomposed third spectrum then appeared not to belong to a distant companion of the close pair, but rather to peculiar emission features, enhanced according to the distortions and deepening of the absorption lines of the first two components. This behaviour may be caused by the distortions of the line profiles of the close binary components due to circumstellar matter (cf. Budding et al. 2005a). We have rejected these solutions as unphysical, despite some of them giving formally a much better fit to the observed spectra than the best solution for a simple binary (as could be expected). The adopted values of these measured radial velocities, listed in Table 3, are shown in Fig. 2 together with theoretical radial velocities corresponding to solution B of Table 4 and the WD procedure referred to in Table 1.

The wide orbit of the close binary with respect to a spectroscopically unseen third companion significantly influences the fit of spectra by disentangling. Because our set of data effectively represents three points only on the corresponding RV-curve with the 1008.8-d period ($\phi' \sim 0.0$, 0.4 and 0.8; cf. Table 3), we converged only the epoch $T'_{\rm per}$ and amplitude K_{1+2} for the wide orbit, while its other parameters were fixed on the values given by Worek (2001). In view of the above discussed uncertainties of the close orbit eccentricity (confirmed by the disentangling), we fixed its eccentricity at $e = 0$, and formally set the periastron longitude $\omega = 90^\circ$, corresponding to a circular orbit. The period and its time-derivative were fixed according to the ephemeris (1), so that the periastron epoch $T_{\rm per}$ (identical with conjunction in this case), the amplitude K_1 and mass ratio q were the only parameters converged for the close circular orbit (together with the epoch and amplitude of the wide orbit).

It is obvious from the principle of the disentangling, which finds the orbital motion by comparing spectra of the same system in different orbital phases (without identification of their lines), that the systemic velocity γ is not directly included in the solution. If γ is of particular interest, for example for comparison with RVs measured by classical methods, it can be found from the disentangled spectra, the wavelength scale of which then corresponds to the RV of the mass-centre of the multiple system.

The disentangled line profile of the Hα line in Fig. 3 is asymmetric. Its central core has RV of approximately -45 km s^{-1}, and

Table 4. Spectroscopic orbital parameters of δ Lib. Solution A is the FOTEL fit to RVs by Tomkin (1978) and Worek (2001); Solution B comes from KOREL disentangling of Hα. Because this method does not provide parameter errors directly, the RVs from B are also solved using FOTEL in C, the difference of values and errors thus gives an estimate of reliability of values given in B. Epochs $T_{\rm per}$ and $T'_{\rm per}$ of the primary minimum and periastron of wide orbit, respectively, are HJD 244 0000, the values of K_1, K_2, K_{1+2} and γ are in km s^{-1}, $q = M_2/M_1 = K_1/K_2$, $q' = K_{1+2}/K_1$ and $c_{K_1{\rm q}}$ is the highest correlation coefficient (cf. text for further discussion).

Solution	A	B	C
		Close orbit	
P		2.32735416 d (fixed)	
$\dot{P}$		3.81549E-10 (fixed)	
$T_{\rm per}$	2960.711 ± 0.004	2960.706	2960.700 ± 0.005
e		0 (fixed)	
ω		90° (fixed)	
K_1	74.1 ± 1.0	81.8	83.8 ± 2.6
K_2	215.3	213.8	218.1
q	0.344 ± 0.006	0.383	0.384 ± 0.015
		Wide orbit	
P'		1008.8 d (fixed)	
$T'_{\rm per}$	3417 ± 31	3480	3315 ± 82
e'		0.28 (fixed)	
ω'		198° (fixed)	
q'	0.067 ± 0.013		0.06 ± 0.04
K_{1+2}	5.0	5.7	5.2
γ	–38.6 ± 0.8		
$c_{K_1{\rm q}}$	0.81		0.93

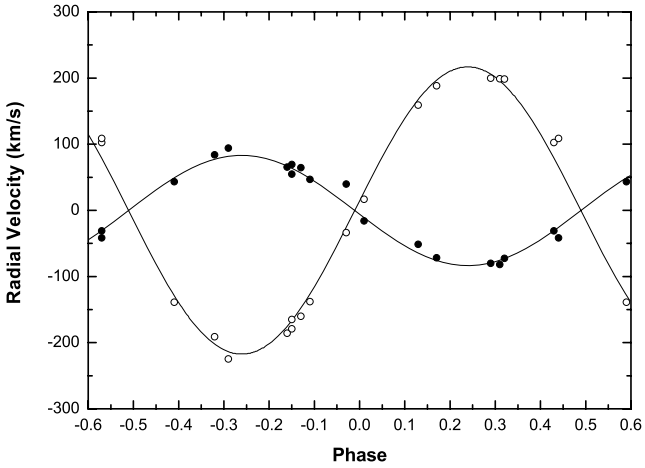


Figure 2. RV fittings for δ Lib.

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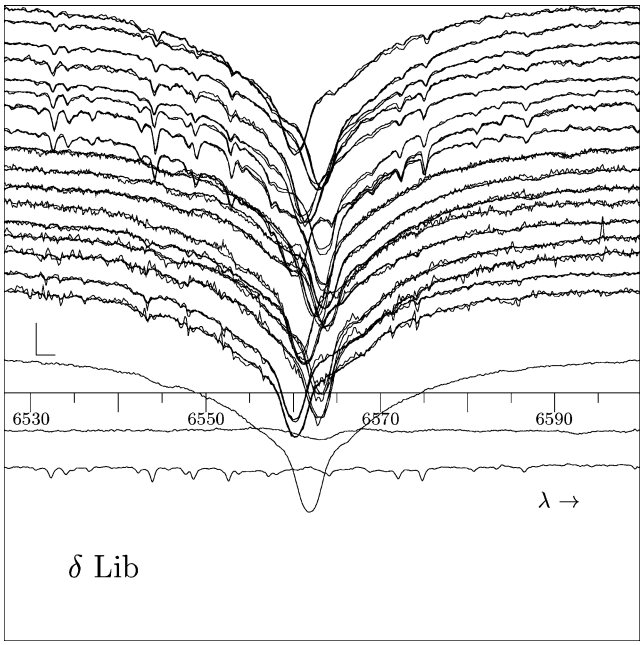


Figure 3. Disentangled Hα lines for δ Lib. The upper 17 curves show the observed line profiles with superimposed KOREL fits. The bottom three lines give the disentangled profile of the primary and secondary component stars and the telluric spectrum.

the velocity calculated by moments of the line profile decreases below −62 km s^{-1} if the wider wings (with a half-width of about 13 Å) are taken into account. It is clear from Fig. 3 that the RV of the secondary component is much more positive. Unevenness of the continuum around this shallow disentangled line hampers a precise RV-measurement, however, a rough estimate gives about +1.5 km s^{-1}.

The wider range of the Ondřejov spectra also involves several weaker lines, the best of which are the Si II lines 6347 and 6371 Å. Because these eight exposures do not cover sufficient phases of the wide orbit, we fixed the orbital parameters to the values of Solution B in Table 4. No traces of the secondary component have been found in these lines, so that for the close orbit only $T_{\rm per}$ and K_1 were converged by disentangling. The results are presented in Table 5 together with the RVs found by KOREL. The systemic velocities of the disentangled profiles (which are not perfectly symmetric), calculated by the moment method, are −45.4 and −46.6 km s^{-1}, respectively (for lines 6347 and 6371 Å), i.e. consistent with the core of Hα of the primary.

Table 5. Orbital parameters and RVs from Si II lines of δ Lib (1996, 1997). (Accuracy is reflected in the number of retained digits.)

	6347	6371			
$T_{\rm per}$	2960.705	2960.712			
K_1	74.50	75.78			
No.	6347	6371	No.	6347	6371
1	−72.49	−72.13	5	−38.20	−39.32
2	52.31	54.89	6	55.56	57.66
3	32.51	32.91	7	60.04	61.43
4	−13.44	−11.77	8	−65.63	−67.88

It is interesting to note that the RV-amplitude of the primary, disentangled from the Si II lines, is consistent with Solution A and previously published values, while from the Hα (Solution B) it is somewhat higher. Because of the large width of Hα, the spectral range for its disentangling has a larger step in RV per bin than for the narrower Si II lines, so that the latter yields higher precision. However, the value of K_1 does not change when we select better resolution in the sampling. The most probable reason for the difference can thus be the difference in orbital line-profile changes between shallow and deep lines (cf. Hadrava & Kubát 2003).

2.3 Discussion of results

Kepler's law, combined with the eclipse solution for the radii, allows us to write

$$3\log R_1 - \log M_1 = 2.6053 - 3\log r_1, \quad (2)$$

but if we use recent empirical data on typical masses and radii for this region of the Main Sequence (e.g. Budding & Demircan 2006), then eliminate the variables, we find a very high mass for the corresponding spectral type. This was noted already by Lazaro et al. (2002), who used evolutionary models in the *T*:*g* plane to deduce that the primary mass cannot be far from 3 $M_\odot$. They adopted 2.85 $M_\odot$ as providing the best match (since *R*, and therefore *g*, can be derived from (2) for a given *M*). The corresponding radius, 3.25 $R_\odot$, is significantly larger than typical MS values. Lazaro et al. (2002) pointed out that the light curves could be consistent with Tomkin's higher masses, but then much larger luminosities would be implied, and the distance to the star becomes too large. It should also be noted that the rotation velocity, if synchronous, as is usual for such close binaries, supports the 3.25 $R_\odot$ radius. Other small changes from the adopted parameters of Lazaro et al. were discussed by Budding et al. (2005a).

Our new spectroscopic analysis leads to the absolute parameters given in Table 6, where the bracketed solution errors give a realistic assessment of formal accuracy for the rather limited set of parameters that the data can resolve. The discrepancy in mass is reduced by the lower amplitude of secondary RV variation, which our results suggest. But Lazaro et al.'s point about the need for further detailed spectroscopy still carries some validity, in view of the limited parameter resolution.

Table 6. Absolute parameters for the δ Lib (ABC) system. Formal errors are indicated by the parenthesized numbers affecting the last retained digit of the solution. (See also Section 2.2 for further discussion of the errors).

Parameter	Value
Period(d)	2.327 352 97
Epoch (HJD)	242 2852.3598
V, $B-V$, $V-J$ (mag)	5.05(2); −0.06(2); −0.01(3)
A_{12} ($R_\odot$)	13.8(2)
$K_{1,2}$ (km s^{-1})	81.8(4.4); 213.8(4.8)
q	0.383(20)
i_{12} (○)	80.7(3)
$M_{1,2,3}$ ($M_\odot$)	4.7(3); 1.8(3); 1.1(2)
$R_{1,2,3}$ ($R_\odot$)	4.15(8); 4.24(8); 1.1(10)
log g (log cgs)	3.73(4); 3.32(4)
$M_{V1,2,3}$ (mag)	0.07(20); 3.04(15); 3.08(20)
$T_{1,2,3}$ (K)	9800(200); 5120(100); 6000(300)
Distance (pc)	99(9)

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3 ASTROMETRY

3.1 Method

Mention was already made of the third body introduced by Worek (2001). This was to account for irregularities found in the primary's RV-curve. Worek's model has the relatively short period for the third orbit as 2.762 yr, that allowed constraints on the third star's mass, depending on the relative inclination of the wide orbit. The sine of this unknown inclination inevitably comes into the scale of RVs when projected in the line of sight and thus limits what can be specified about absolute parameters. But Worek's solution could, in principle, be checked from *Hipparcos* observations that covered a ∼3 yr time interval, since the scale of the implied orbital displacements should be of the order of 5 mas (given that Worek's $a \sin i$ is $\sim 7 \times 10^7$ km), i.e. significantly greater than the 1–2 mas typical accuracies of the satellite's stellar positions. The possibility of combining astrometric data with other time-dependent information offers an opportunity to investigate the complete geometrical arrangement of the component stars and separate out corresponding absolute parameters.

Ambiguities are possible in bringing RV data into a conventional astrometric framework, since the 'longitude of periastron' for RVs is normally measured from the plane of the sky, and not the line of nodes in the equator, as in the standard three-dimensional specification for astrometric double stars. If we use the local plane of the sky as the reference, the angle Ω should then refer the intersection (nodal) line to the equatorial coordinate system. Clearly, this angle has no effect on RVs, though it will be a parameter for astrometric data.

To quantify this, let us transform from the natural reference frame for the wide (assumed Keplerian) orbit, in which the eclipsing binary is at $\xi\ (= r\cos\nu)$, $\eta\ (= r\sin\nu)$, $\zeta (= 0)$, where r is the radius from the system centre of gravity (origin) and ν is the true anomaly; to a sky reference frame. We may visualize an x-axis increasing to the left as with conventional RA (α), y increasing to upwards like declination (δ) and z increasing along the outwards line of sight.[2] A positive rotation about the z-axis, from this viewpoint, is seen as clockwise on the sky. To do the transformation, rotate first about the ζ-axis by $-\omega$, where the periastron longitude ω can be simply regarded as adding to ν regarding the binary's angular displacement from the new ξ-axis of reference $(= x)$. Then, to transfer from the ζ-axis to the outwards line of sight, setting the x-axis as that of the ascending node, rotate back about this x-axis by the complement of the inclination, i.e. $-(\pi - i)$. Applying the corresponding rotation matrices to the vector $\boldsymbol{\xi}$, we find

$$\begin{aligned} x &= r\cos(\nu+\omega),\\ y &= -r\sin(\nu+\omega)\cos i,\\ z &= r\sin(\nu+\omega)\sin i. \end{aligned} \tag{3}$$

With this geometry in mind, it follows (cf. Irwin 1952) that the time taken for light from a close eclipsing binary in orbit around the centre of gravity of a tertiary system to reach the Earth contains the variable component $\tau = z'/c$, c being the velocity of light and z' given by

$$z' = a\sin i\left[\frac{1-e^2}{1+e\cos\nu}\sin(\nu+\omega) + e\sin\omega\right]. \tag{4}$$

The symbols here have their generally conventional meanings, but Irwin refers the light travel time (LTT) effect to the centre of the ellipse (rather than the centre of gravity, or focus); thus adding the term $ae\sin i\sin\omega$ to z, to give z'.

By differentiating this expression with respect to time, replacing ω by $\omega - \pi/2$, e by $e/2$ and neglecting higher powers than the first in e, we will see that the shape of the LTT curve resembles that of a RV ($\dot{z}$) curve having half the eccentricity and ω decreased by $\pi/2$. On this basis, we have essentially the same determinacy problem as applies to RV-curves: the scale contains a and $\sin i$ as an inseparable product, while the nodal angle Ω does not even enter the equations of condition.

The sky reference frame x, y, z presented above pictured x as increasing in the same way as RA: in practice, there is an additional degree of freedom about the line of sight, allowing x to be at an arbitrary orientation θ, say, to the E–W direction. To move from the orbit on the sky to the actual equatorial reference frame x, y, z; say, we thus need to rotate back through θ, so that $X = x\cos\theta - y\sin\theta$, $Y = x\sin\theta + y\cos\theta$. Since $Z = z$, the LTT and RV discussions are not affected. The convention normally adopted for position angle on the sky (Ω) is clockwise around from the direction of increasing declination; thus $\Omega = 3\pi/2 + \theta$, hence

$$X = \frac{-a(1-e^2)}{1+e\cos\nu}\left[\cos(\nu+\omega)\sin\Omega - \sin(\nu+\omega)\cos\Omega\cos i\right], \tag{5}$$

$$Y = \frac{a(1-e^2)}{1+e\cos\nu}\left[\cos(\nu+\omega)\cos\Omega + \sin(\nu+\omega)\sin\Omega\cos i\right], \tag{6}$$

$$Z = \frac{a(1-e^2)}{1+e\cos\nu}\sin(\nu+\omega)\sin i. \tag{7}$$

Hence, for example, if ω and $\Omega = 0$ and $i = \pi/2$, the ellipse aligns with its periastron apse parallel to increasing declination, if $\Omega = \pi/2$ it is parallel to decreasing RA, and so on. Alternative arrangements of the signs and trigonometric functions in the foregoing equations are possible, of course, depending on adopted conventions: the important point is that the investigator consistently relates observed data to the adopted geometrical arrangement of an applied model.[3]

The natural reference frame for the wide orbit also allows fairly simple expressions for the time derivatives as

$$\begin{aligned} \dot{\xi} &= \frac{-2a\sin\nu}{P\sqrt{1-e^2}}\\ \dot{\eta} &= \frac{2a(\cos\nu+e)}{P\sqrt{1-e^2}}\\ \dot{\zeta} &= 0. \end{aligned} \tag{8}$$

We may then perform the same transformations on these velocities to obtain

$$\dot{X} = k(U_1\sin\Omega + U_2\cos\Omega\cos i) \tag{9}$$

$$\dot{Y} = -k(U_1\sin\Omega - U_2\cos\Omega\cos i) \tag{10}$$

$$\dot{Z} = kU_2\sin i, \tag{11}$$

where $U_1 = \sin(\nu+\omega) + e\sin\omega$, $U_2 = \cos(\nu+\omega) + e\cos\omega$ and $k = 2\pi a/(P\sqrt{1-e^2})$, P being the orbital period.

The measurements of *Hipparcos* are not directly in the equatorial system, but concern great circle abscissal measures p, say. The variations of p in dependence on small changes of a given position of a star X_1, Y_1, Z_1, say, and its adopted motions through the observation

[2] Local measures in the α direction should, of course, be multiplied by sec δ to produce the corresponding changes in α.

[3] Ribas, Arenou & Guinan (2002), for example, measure Ω in the opposite sense to that given here.

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period from which the *Hipparcos* solutions are derived, $\dot{X}_1$, $\dot{Y}_1$, $\dot{Z}_1$, are given (in principle) as

$$\Delta p = \frac{\partial p}{\partial X_1}\Delta X_1 + \frac{\partial p}{\partial Y_1}\Delta Y_1 + \frac{\partial p}{\partial Z_1}\Delta Z_1 + \frac{\partial p}{\partial \dot{X}_1}\Delta \dot{X}_1 + \frac{\partial p}{\partial \dot{Y}_1}\Delta \dot{Y}_1 + \frac{\partial p}{\partial \dot{Z}_1}\Delta \dot{Z}_1. \quad (12)$$

In practice, there is considerable difference in effect, for a given spatial displacement in the plane of the sky and the scale of components ΔX_1, ΔY_1 and ΔZ_1. In other words, the accuracy with which *Hipparcos* astrometry can fix the X and Y co-ordinates ($\sim 10^{-8}$ in angular measure) is a number of orders greater than that with which it can fix the parallax (as are stellar distance orders greater than the astronomical unit). Hence, although a possible correction to the parallax is retained in practical applications of equation (12), we need not concern ourselves with the effects of Z displacements that might arise from orbital motion of a wide binary, such as we have been evaluating, as far as *Hipparcos* data are concerned. For practical purposes, equation (12) reduces to

$$\Delta p = \frac{\partial p}{\partial X_1}\Delta X_1 + \frac{\partial p}{\partial Y_1}\Delta Y_1 + \frac{\partial p}{\partial \Pi}\Delta \Pi_1 + \frac{\partial p}{\partial \dot{X}_1}\Delta \dot{X}_1 + \frac{\partial p}{\partial \dot{Y}_1}\Delta \dot{Y}_1, \quad (13)$$

where Π_1 is the derived parallax, and ΔX_1, ΔY_1, $\Delta \dot{X}_1$ and $\Delta \dot{Y}_1$ can combine the effects of systematic displacements due to an orbital motion, together with constant errors in the originally assigned position X_1, Y_1 and proper motions $\dot{X}_1$, $\dot{Y}_1$ on the basis of neglect of such orbital effects in the published *Hipparcos* position, parallax and proper motion values. In mathematical form, then, we have

$$\Delta p = \frac{\partial p}{\partial X_1}(X + \delta X_1) + \frac{\partial p}{\partial Y_1}(Y + \delta Y_1) + \frac{\partial p}{\partial \Pi}\Delta \Pi_1 + \frac{\partial p}{\partial \dot{X}_1}\Delta \dot{X}_1 + \frac{\partial p}{\partial \dot{Y}_1}\Delta \dot{Y}_1, \quad (14)$$

with X and Y being supplied for a given model of the orbit, in dependence on the parameters a, e, i, ω, Ω and P through equations (5) and (6). To calculate where the star would be at any particular time, we also need the parameter T_0 that gives the epoch of periastron passage. Here, the introduced small constants δX_1, δY_1 make up the resultant measured differences $\Delta X_1 = X + \delta X_1$, $\Delta Y_1 = Y + \delta Y_1$. In principle, $\Delta \Pi_1 = Z\Pi/a + \delta \Pi_1$, where Z is in mas like X and Y, and $a = 2.06 \times 10^8$, but the term in Z would generally be so small compared with available accuracies as to be negligible.

Note also that if the orbital period were large in comparison to the period during which the *Hipparcos* observations were gathered, the mean annual proper motions derived by *Hipparcos* would be affected by such a binary motion, leading to values that could be systematically different from those averaged from terrestrial observations over many years. The deviations, for binaries of period a decade or two, could even be non-linear over the 3–4 yr of the *Hipparcos* survey. In some known examples of such binary motion, this led the catalogue compilers to the introduction of extra non-linear terms in (12), i.e. seven or even nine term solutions (cf. van Leeuwen & Evans 1998). This is not the case, however, for an orbit like that of Worek, whose period is less than that of the survey. The instantaneous angular velocities, as seen from Earth, due to this orbit (equations 9–10), are sometimes a notable fraction of the systematic proper motion, but, over the ∼3 survey years, if evenly distributed in time, would just add to the 'noise' for the determination of the mean proper motions. If correct mean proper motions were thus determined, the orbital displacements would separate out as a discernible Keplerian motion superposed on that of the system centre of gravity. If the data were unevenly distributed, however, as appears the case for δ Lib, then some systematic error of the *Hipparcos* mean proper motions could be expected. A small linear correction $\Delta \dot{X}_1$, $\Delta \dot{Y}_1$, arising from this cause, should have the effect of reducing the observational scatter in a revised solution that took into account orbital effects.

The five extra terms in equation (14), i.e. $\delta X_1, \delta Y_1, \Delta \Pi_1, \Delta \dot{X}_1$ and $\Delta \dot{Y}_1$, add to the seven orbital parameters mentioned already to make up a total of 12 unknown parameters for an optimization fitting procedure. The five partial derivatives that can be denoted $F_i (i = 1, 5)$, in equation (14), were supplied as numerical tabular values for each measured *Hipparcos* position. [F_1 and F_2 can be regarded here simply as $\sin\theta$ and $\cos\theta$, where θ is a position angle of a reference great circle (cf. van Leeuwen and Evans 1998)]. We can thus use equation (14) and each *Hipparcos* measurement as an equation of condition to optimize the fit of the parameters for a given model. Essentially, (14) allows χ^2 to be calculated by a nested subroutine within an ILOT program environment. As with many optimization problems, however, it is better, if possible, to split the determinacy between the astrometric and other observations (e.g. LTT data) that involve the same parameters. If the model is completely wrong, we would expect the run of residuals Δp to be worse than for the no-orbit solution. Tests were, in fact, made by the *Hipparcos* data reduction consortia for numerous stars of suspected duplicity (including δ Lib), but often with no significant improvement over the original solutions.

3.2 Worek's third star

To proceed with an application, first consider the case of no errors of measurement. All measured deviations Δp would then arise solely from ordered changes of position, neglected in the no-orbit solution of the catalogue. In the case where expected changes of position due to a binary motion are significantly greater than the real errors of measurement, this assumption is a tolerable first approximation. We can then write $\Delta X_1 = F_1 \Delta p$ and $\Delta Y_1 = F_2 \Delta p$, to examine the fitting in the orbital plane.

Worek's spectroscopy-derived parameters are $a \sin i = 5.3$ mas, $e = 0.28$, $\omega = 198^\circ$, $P = 2.762$ yr and $T_0 = 1983.24$. The zero of the *Hipparcos* orbit number time-scale corresponds to 1989.847, which means that the mean anomaly at *Hipparcos* time zero is 141°. A cursory examination of the *Hipparcos* Intermediate Data Search Facility (IDSF) (http://www.rssd.esa.int/*Hipparcos*/apps/PlotHipi.html) does suggest a slight quasi-sinusoidal variation on a ∼2.8 yr time base. Extraction of the relevant data file from this source and applying the reasoning of the last paragraph show the observation points rather elongated about an axis at position angle ∼145°–325°, suggesting the inclination is not too far from 90° as surmised by Worek. Fig. 4 shows this distribution, together with a plotting of a set of corresponding orbital positions, taking the orbit to be coplanar with the eclipsing system, $\Omega = 327^\circ$, the other five orbital parameters from Worek, and displacements of the focus $\delta X_1 = -1.7$ mas, $\delta Y_1 = -2.5$ mas.

Although the orbit seems feasible, since the plotted ellipse matches the general asymmetric distribution of observations, it is, in fact, significantly non-optimal. The rms error is more than twice the ∼1.5 mas that could be expected for normal *Hipparcos* positions. Assuming a coplanar inclination and adopting all the other parameters from Worek, minimization of the sums of squares of errors by changing only Ω moves the ellipse major axis away from

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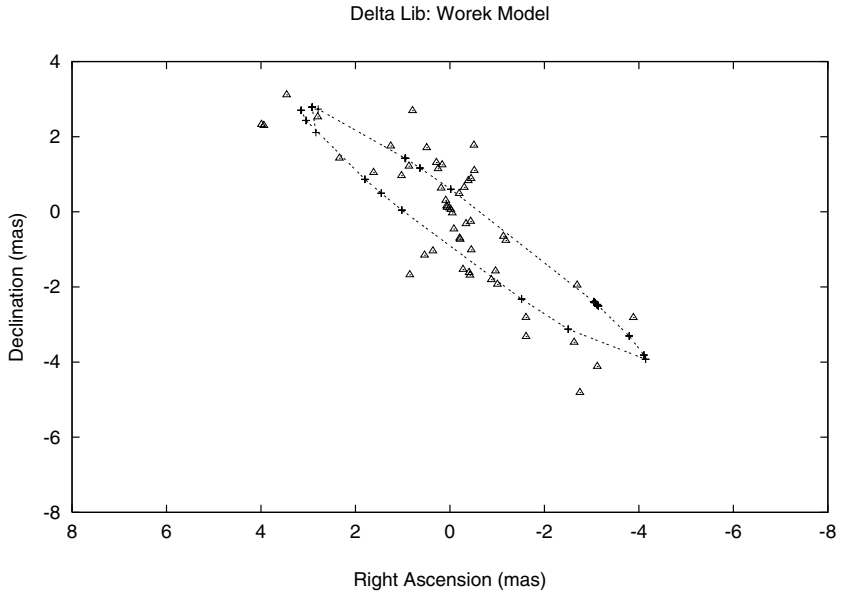


Figure 4. Plot of *Hipparcos* angular measurement components in $\alpha \cos \delta$ and δ (in mas) on the assumption of displacements due to Worek's (2001) 2.762 yr period third body.

the apparent X, Y correlation axis, though without improving the rms error appreciably. The asymmetry in time distribution of the observation points in relation to the orbit is noteworthy here. This comes partly from an inherently inhomogeneous sampling and also the significant eccentricity implying that, in general, periastron positions would have less likelihood to be observed. This is perhaps related to why an orbital solution was missed in the original reductions, though one or two more conspicuously anomalous of the original residuals are located towards the periastron region at the lower right of the diagram.

It was mentioned that the preliminary IDSF examination indicated a period of ∼2.8 yr: a little, but notably, more than Worek's value. Such an increased period also improves the phasing, assuming the correctness of Worek's epoch. In Fig. 5, we show the run of the model against *Hipparcos* residuals 'corrected' for slight changes of system centre position, parallax and proper motions and a slightly increased period. The scatter here now seems at least as good as the distribution of the original residuals about the no-orbit solution. We present a corresponding set of adopted parameters in Table 7.

Budding et al. (2005a) did not find any clear solution for the Worek model orbit in the *Hipparcos* residuals. It is true that Worek's RV-curve for the third orbit uses only eight points, of which only four were new and the other four involved indirect inferences from sometimes very old data. Nevertheless, there seems no good reason to doubt the high quality of the four new Kitt Peak National Observatory measures, which certainly appear to identify a RV amplitude

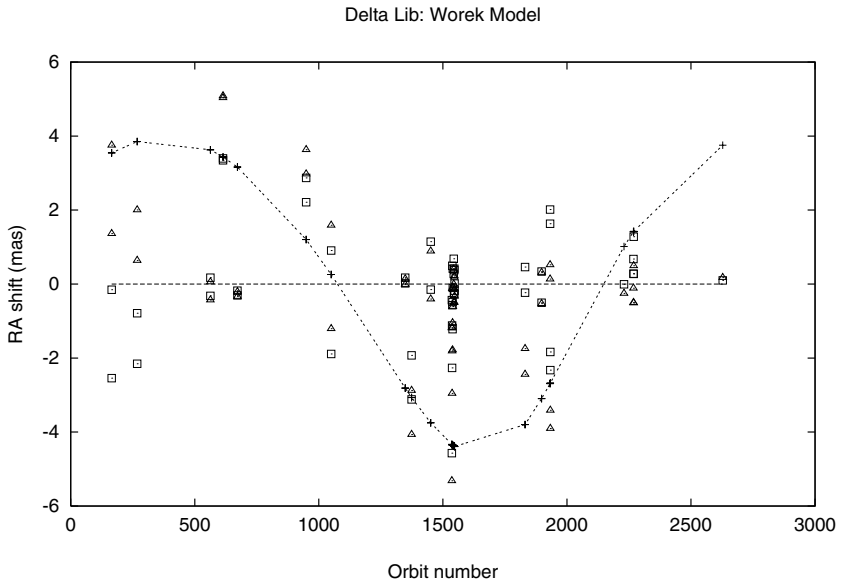


Figure 5. Plot of *Hipparcos* data trial fit to an orbit of δ Lib about a barycentre with a third body. We compare a Worek-like orbit, projected in the RA direction (triangles and curve), with the no-orbit solution of the catalogue (squares and straight line).

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Table 7. Astrometric elements for δ Lib AB-C system.

Parameter	Value
P	2.91 ± 0.35 yr
a	4.8 ± 0.2 mas
e	0.28
ω	198°
i	$113° \pm 10°$
Ω	$287° \pm 20°$
T_0	0.79 yr
Π	10.10 ± 0.9 mas
$\Delta\alpha \cos\delta$	−1.4 mas
$\Delta\delta$	−0.8 mas
$\Delta\mu_\alpha \cos\delta$	2.9 mas yr^{-1}
$\Delta\mu_\delta$	1.9 mas yr^{-1}
$\mu_\alpha \cos\delta$ (ad)	-66.2 ± 0.8 mas yr^{-1}
μ_δ (ad)	-3.6 ± 0.8 mas yr^{-1}
χ^2/ν	1.04
ΔX	2.12 mas

Notes: This table lists our adopted best-fitting astrometric elements. The period P is 10 per cent – significantly–longer than the 2.761 yr value of Worek (2001). The semimajor axis is reduced by 15 per cent on that of Worek, but we have decreased the parallax by a corresponding amount. Eccentricity e and longitude of periastron ω values come from Worek (2001), but the inclination i and nodal angle Ω values are new.

Regarding the parameter T_0: the zero point of the *Hipparcos* orbit number (time) scale corresponds to 1989.847. Using Worek's epoch of 1983.24, and period of 2.76 yr, δ Lib was at phase zero at 1988.76 (= $2P$ later), i.e. 1.08 yr before the zero of the *Hipparcos* times (T_0). If the period is increased, as we have done, the last periastron before the start of the *Hipparcos* reckoning would be later. The effect of increasing P is then to decrease T_0 to the adopted value of Table 7, with the same error as that of the period.

The small centroid shifts $\Delta\alpha \cos\delta$ and $\Delta\delta$ are within the *Hipparcos* errors of the catalogue position, and the proper motion corrections $\Delta\mu_\alpha \cos\delta$ and $\Delta\mu_\delta$ are similarly small. We also compared the *Hipparcos* proper motions with linear averages from listed historic positional measurements, and found them to be essentially similar, within the understandably larger errors of the latter. More significant, perhaps, is the good agreement between *Hipparcos* and radio VLA/VLA+PT determinations (Boboltz et al. 2003), whose average values we adopt in Table 7. But, here the relatively large errors of the radio measurements, collected for over 22 yr, should be noted. Leaving out HD50896-N (for reasons explained by Boboltz et al.), the proper motion errors for δ Lib are 50 and 90 per cent greater, in RA and Dec., respectively, than the average of 17 listed radio stars, and comparable to the proper motion corrections we find.

The value of χ^2/ν indicates a statistically probable result with the feasible SD error of a single measurement ΔX of around 2 mas.

of at least 5 km s^{-1} to significantly better than 10 per cent of this value. In turn, this leads to a mass function f [$= P(K/29.79)^3$] of ∼0.015.

Worek adopted a rather higher $\sin i$ than ours, together with Tomkin's (1978) higher masses ($M_1 + M_2 = 6.6\,M_\odot$) to find a third mass of about $0.95\,M_\odot$. With a somewhat longer period and lower $\sin i$ ($i = 113°$) coming from optimal fittings to the astrometry, together with Tomkin's (1978) masses, we could raise this mass to $1.1\,M_\odot$. This comes sufficiently close to the mass estimate of Section 2 to make Worek's star a feasible explanation of the photometric third light, if some of the foregoing parameter values can be relaxed. The inclination appears rather sensitively in the mass derivation, and a value of $i_{12,3} = 120°$ is within the parameter-interdependent error of the astrometric estimation. The residual uncertainty definitely gives a strong motivation towards more refined new observations here.

3.3 LTT effects

Budding et al. (2005a) fitted a light time effect associated with a third body in an eccentric orbit (equation 4) to the O–C residual safer removing the main parabolic term and giving extra weight to the photoelectrically observed minima. The parameters of this fit were $a \sin i = 1.52$ au, $P = 54.73$ yr,[4] $e = 0.49$, *and* $\omega = 317°$. The corresponding mass function is only 1.18×10^{-3}. Although this orbital effect improved the total goodness of fit parameter χ^2, it was only a 10 per cent improvement over a straight line fit to the residuals and sensitive to the weights given to the relatively few photoelectric times of minima. Thus, whilst of interest, it could not be considered decisive.

Budding et al. (2005a) also tried a check of the *Hipparcos* astrometric data for this LTT result, in which the period, projected semimajor axis, eccentricity and longitude values were taken from the preceding tentative O–C fitting and inclination and nodal angle values were optimized, also allowing for a mean shift in the centroid of the orbit from the centroidal position given for δ Lib in the *Hipparcos* catalogue. This approach appears essentially similar to that of Ribas et al. (2002) for R CMa, and like that solution, the optimizing one found for δ Lib placed the bright star rather close to periastron and with high inclination ($i = 86.5$, $\Omega = 279.^\circ 2$). As with R CMa, a relatively large shift of the centroid in RA (19.0 mas) was also implied, but the mass that goes with this inclination and (corrected) mass function would only be $\sim 0.39\,M_\odot$. In order for the LTT effect to match the sought third star mass, the inclination would have to yield $\sin i \sim 0.33$, implying a greater movement on the sky than observed by *Hipparcos*, unless there happened to be some very fortuitous combination of proper motions. But, as already noted in connection with the astrometry of the Worek model orbit, there is not a large discrepancy between the *Hipparcos* and historic mean proper motions.

In view of the feasible astrometric fitting of the Worek orbit of the present paper, we rechecked the O–C data for an effect on this time-scale. We show a typical result in Fig. 6. This plot illustrates the underlying difficulty of the approach, at least as much as it lends support to Worek's model. Set against the general scale of error of the data, this particular optimization problem looks ill-posed. There are many local shallow minima and no clearly decisive outcome.

The possibility of more than one local minimum of χ^2 is also borne out in the previous long period fitting to the astrometry. Budding et al. (2005a) concluded that their tentative long-period orbit may have to wait for confirmation until the precision astrometry of *GAIA* becomes available. The rather different perspective of the present paper is that, regardless of support for either a long or a short period wide orbit from LTT data alone, the mutual support of photometry, spectroscopy and astrometry regarding the ∼2.9 yr period is noteworthy.

4 CONCLUSIONS

In the present study, we have checked the previous photometric curve-fittings of Budding et al. (2005a) using some other well-tested light-curve fitting programs and found a reasonably close agreement. In particular, we confirm that the photometric evidence, when examined carefully, points to the existence of a third star: probably of mid- to late-F spectral type and likely to be around $1.1\,M_\odot$.

[4] Note that this period in the paper of Budding et al. (2005a) is erroneous by a factor of the numerical value of the period in days.

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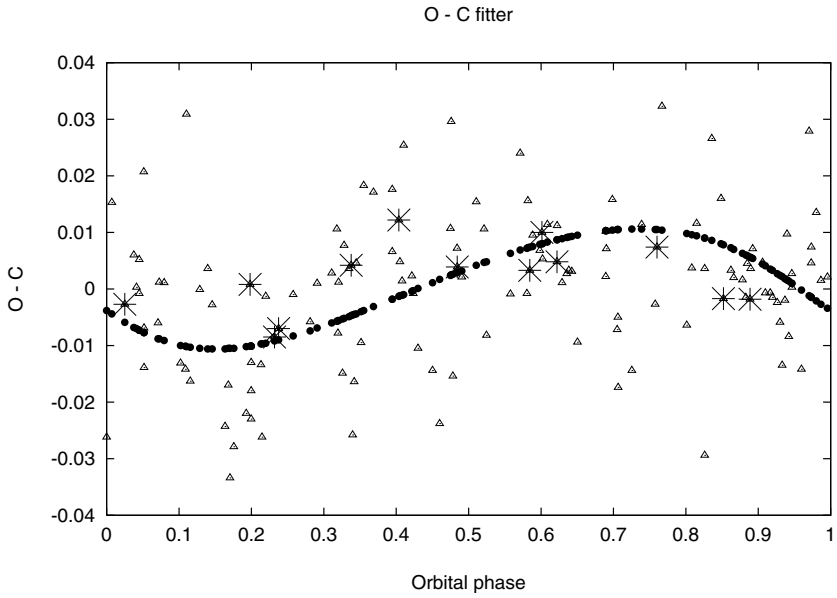


Figure 6. Fit of residuals from Kreiner et al.'s (2001) O–C data on δ Lib, after the removal of the main parabolic term, to an eccentric orbit similar to that of Worek (1998). Photoelectric times of minima are shown as large stars.

Using new RV data, we have recomputed absolute parameters for the eclipsing binary components (Table 6). The difficulties of analysis and interpretation reported by Lazaro et al. are likely to be at least partly resolved by inclusion of the third component, which then tends to result in a somewhat hotter primary star. Our preliminary independent high-dispersion spectroscopy suggests a higher mass ratio, and heavier masses consistent with those by Tomkin (1978), but more new data are needed to confirm this point.

Analysis of times of minimum light (O–C effects) broadly confirms the classical Algol status of δ Lib. The rate of period increase $\Delta P/P \sim 4.44^{-10}$ suggests, according to the Case B formulae of Murad & Budding (1984), a rather low relative surface density of the loser's envelope ($\eta < \sim 0.05$), even with an initial mass not much greater than half the present total mass (i.e. $\sim 2\,M_{\odot}$), but this inference is compromised by the approximations used–originally applied to Algols of higher mass than δ Lib–and also the role of the third body in complicating the angular momentum exchange history. Subsequent more detailed spectroscopy may enable a closer check on the secondary's physical behaviour.

A procedure for the analysis of *Hipparcos* Intermediate Astrometric Data has been detailed that may be applied to objects similar to δ Lib in the future. Its application in the present case indicates that there is a third star similar to the one identified by Worek (2001) from RV data, although with slightly revised parameters. With the third orbit coplanar with the eclipsing system, Worek's solution yields the astrometric position angle of the axis to be roughly NW–SE on the sky and fairly small ($\sim$2 mas), displacements of the focus from the *Hipparcos* centroid.

An improved overall concurrence comes from reducing the inclination of wide orbit to around -70° and relaxing its period to $\sim$2.9 yr: somewhat longer than Worek's, but producing a fitting that is at least as good as the no-orbit solution of the *Hipparcos* Catalogue. The astrometric third star mass then agrees with the photometric one within realistic solution errors ($1.1 \pm 0.1\,M_{\odot}$). The mean separation of this companion from the close past turns out to be $\sim$4 au. The previously conjectured (Budding et al. 2005a) much wider third orbit is not convincingly supported by the combined data, and fails to pass an 'Okham's razor' test of resolving all the data anomalies. Worek's star does much better on this, although the present orbital parameters still lack sufficient precision.

The bright classical Algol δ Lib thus invites further attention using the power of combined astronomical techniques. The extensive background of data shows intriguing, but still insufficiently well-resolved effects. It continues to offer challenges in the detailed confrontation of theory and observations within stellar astronomy.

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